PS Algorithms and Data Structures 2026

Task sheet 9

Task 25

A logistics company operates a set of logistics centers L and wants to supply a set of locations O. Between these sites (both between centers and locations, but also between two centers or two locations), there are connections (e.g., roads), each having a known positive distance. Keep in mind that these connections do not have to be symmetrical (e.g. one-way streets). For each location, the nearest logistics center (measured by the total distance over the connections) is to be determined.

1. Describe how this problem can be modeled as a graph problem.
2. Based on your modeling from part 1, develop an algorithm that finds for each location $o \in O$ the logistics center $l^* \in L$ such that the distance $\delta(l^*, o)$ is minimal. The algorithm should have a runtime of $O((|E| + |V|) \log |V|)$ for a graph $G = (V, E)$.

Task 26

Kruskal's algorithm calculates a minimum spanning tree by specifying a start node.

- Show that the minimum spanning tree calculated by Kruskal's algorithm is not unique in general.
- What changes must be made to Kruskal's algorithm to obtain a maximum spanning tree?

Task 27

Consider an undirected, weighted graph $G = (V, E)$. We define the *heaviest edge* of a cut $(C, V \setminus C)$ analogously to the *easiest edge* defined in the lecture. Show that G has a unique *maximal* spanning tree if for each cut $(C, V \setminus C)$ of G there exists a unique *heaviest edge* crossing $(C, V \setminus C)$.