

PS Algorithms and Data Structures 2026

Task sheet 5

Task 13

Given a sequence S of insert and delete operations on a binary search tree, let S' be another such sequence that differs from S only by swapping two consecutive delete operations. Prove or disprove: In general, S and S' result in the same binary search tree.

Task 14

Show that Heap-Sort has a runtime of $\Omega(n \log n)$ if the input array A has the following structure:

Let $n = 2^k - 1$ with $k \in \mathbb{N}$, $k \geq 1$ be the length of the array. The first $\lfloor \frac{n}{2} \rfloor$ entries consist of the numbers $\lfloor \frac{n}{2} \rfloor + 1$ through n in arbitrary order, while the last $\lfloor \frac{n}{2} \rfloor$ entries consist of the numbers 1 through $\lfloor \frac{n}{2} \rfloor$ in ascending order.

Note: It is not enough to argue that Max-Heapify generally has runtime $\Omega(\log n)$.

Task 15

Execute the following operation sequence for an AVL tree, performing the necessary rotations:

- | | | | |
|---------------|---------------|---------------|----------------|
| 1. insert(30) | 4. insert(40) | 7. insert(12) | 10. insert(1) |
| 2. insert(20) | 5. insert(25) | 8. insert(28) | 11. delete(25) |
| 3. insert(10) | 6. insert(27) | 9. insert(5) | 12. delete(20) |